

STRONG DECAYS OF $Q\bar{Q}$ MESONS

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Abstract

We present a detailed study of the two-body strong decays of light mesons. Both the space part and the spin-flavor-color part of the wave functions are generated algebraically and closed forms are obtained for all decays. Experimental deviations from our systematics are seen to be suggestive of both missing mesons and exotic QCD configurations.

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1 Introduction

In the last 30 years, a considerable amount of experimental information has been accumulated on the spectroscopy of hadrons. In addition to masses, electromagnetic, weak and strong decay widths have been measured to the extent that the most recent compilation [1] contains many pages and a voluminous listing of decay modes of mesons and baryons. While the earlier data were analyzed in terms of symmetries, most notably the “eightfold” $SU(3)$ flavor symmetry of Gell–Mann [2] and Ne’eman [3], in recent years there has been a tendency to analyze those data in terms of the non–relativistic [4] or relativized [5] quark model, in which a Schrödinger–like equation is solved with some two–body quark–quark interaction V_{ij} .

We have recently initiated a reanalysis of hadronic spectroscopy by reintroducing the concept of symmetry, now enlarged from flavor $SU_f(3)$ [2, 3] or flavor–spin $SU_{fs}(6) \supset SU_f(3) \otimes SU_s(2)$ [6] to include the space part of the hadronic wave function [7, 8]. We use an algebra \mathcal{G} that generates all excitations, $\mathcal{G} = \mathcal{R} \otimes \mathcal{G}_s \otimes \mathcal{G}_f \otimes \mathcal{G}_c$, where \mathcal{R} is the space algebra, \mathcal{G}_s the spin algebra, \mathcal{G}_f the flavor algebra and \mathcal{G}_c the color algebra. The spin–flavor–color part is the same as in earlier analyses. The novel part is the use of a spectrum generating algebra for the space part which allows us to construct all states starting from the ground state. Hence we are able to put into a single representation of the algebra \mathcal{G} all hadron states, including orbital and radial excitations and not just states with a given value of the orbital and radial quantum number.

The purpose of our reanalysis is twofold:

- (i) we want to condense the information contained in the extensive tabulations of Ref.[1] into a small set of parameters;
- (ii) more importantly, we want to identify “new” physics, if any, which emerges from the data set. If a data point deviates considerably from the parameterization of (i) (i.e. the symmetry is badly broken), and this deviation cannot be explained in a reasonable way, we take it as indication that new physics is at play.

By new physics, in the present context, we mean unconventional configuration of

quarks and gluons. These unconventional configurations are in many cases demanded by QCD and it would be quite surprising if they are not found (or a reasonable explanation is not found for their absence). An example of these configurations are, in meson spectroscopy, purely gluonic states (glueballs). Unless a somewhat model independent framework is set up, against which the data can be compared, it would not be possible to identify uniquely those states which occur in a region of overlapping resonances.

In this article, we analyze strong two-body decays of $q\bar{q}$ mesons, and show that the known decay widths can be reasonably well summarized by a transition operator with only two parameters. This parameterization provides a description of the known widths within a factor of two or better, and thus we believe that it can make predictions of unknown widths with this accuracy. It would be of great interest to check these predictions by measuring some of the unknown widths. We find it particularly important that, by using space symmetries, the results for the decay widths can be written in a transparent way which isolates the various contributions: kinematic factors, spin-flavor part, and space part (form factors). The space symmetry can be viewed as a way to generate the hadronic form factors in a consistent way.

2 Method of Calculation

In this article, we consider all two-body decays of $q\bar{q}$ mesons where one of the emitted mesons is a pseudoscalar:

$$M \longrightarrow M' + M''. \quad (1)$$

In order to compute the strong decays (1), we need the meson wave functions and the form of the transition operator.

A. Meson wave function

We use an algebraic construction of the wave functions. The spectrum generating algebra (SGA) is

$$\mathcal{G} = \mathcal{R} \otimes \mathcal{G}_s \otimes \mathcal{G}_f \otimes \mathcal{G}_c, \quad (2)$$

where \mathcal{R} is the space part, \mathcal{G}_s the spin part, \mathcal{G}_f the flavor part and \mathcal{G}_c the color part. Following Refs. [7, 8] we take

$$\mathcal{R} \equiv U(4), \quad \mathcal{G}_s \equiv SU_s(2), \quad \mathcal{G}_f \equiv SU_f(3), \quad \mathcal{G}_c \equiv SU_c(3), \quad (3)$$

and consider only light quarks, u, d, s . The corresponding wave functions are of the type

$$\psi = \psi_{\mathcal{R}} \otimes \psi_s \otimes \psi_f \otimes \psi_c. \quad (4)$$

The color part of ψ does not play any role for $q\bar{q}$ mesons, as long as we construct color singlet states. The flavor part can be written either explicitly as $|q_i\bar{q}_j\rangle$ for quark of flavor i and antiquark of flavor j , or as a $SU_f(3)$ wave function

$$\left| \begin{array}{ccccccc} SU_f(3) & \supset & SU_I(2) & \otimes & U_Y(1) & \supset & SO_I(2) \otimes U_Y(1) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ (\lambda, \mu) & & I & & Y & & I_3 \end{array} \right\rangle. \quad (5)$$

The space and spin parts are coupled to total angular momentum J , and written as

$$|N, f, L, S, J, M_J\rangle = \sum_{M_S, M_L} \langle L, M_L, S, M_S | J, M_J \rangle |S, M_S\rangle |f, L, M_L\rangle. \quad (6)$$

Here f is a radial quantum number, N labels the irreducible representation of $U(4)$ (i.e. the Hilbert space on which calculations are performed) and L, S, J, M_J have obvious meaning.

In order to study the decay widths in as much as possible model independent way, we consider two limiting cases of wave functions corresponding to the two dynamic symmetries of $U(4)$

$$\begin{array}{ccc} & U(3) & (I), \\ \nearrow & & \searrow \\ U(4) & & SO(3) \supset SO(2) \\ \searrow & & \nearrow \\ & SO(4) & (II). \end{array} \quad (7)$$

The first corresponds to the harmonic-oscillator quark model [4], the second to a string-like situation [7, 8]. The wave functions of chain (I) are labelled by

$$|N, n, L, M_L\rangle, \quad (8)$$

where n is the harmonic oscillator principal quantum number. The wave functions of chain (II) are labelled by

$$|N, v, L, M_L\rangle, \quad (9)$$

where v is the vibrational quantum number. The wave functions (8) and (9) differ in their radial part but have the same angular part, since $SO(3)$ is a common subalgebra of both $U(3)$ and $SO(4)$. The quantum number assignments of some mesons are shown in Table I. Another possible notation is the spectroscopic notation $^{2S+1}L_J$.

B. Form of the transition operator

We assume that the meson M'' is emitted by quark q_i or antiquark \bar{q}_j in meson M which then changes to meson M' . In this picture, the quark contribution to the transition operator is of the form [5]

$$\mathcal{H}' = X_{q_i q'_i}^{M''} \left[g (\boldsymbol{\sigma}_i \cdot \mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{r}_i} + h (\boldsymbol{\sigma}_i \cdot \mathbf{p}_i) e^{-i\mathbf{k} \cdot \mathbf{r}_i} \right], \quad (10)$$

where \mathbf{k} is the momentum of the emitted meson, and $X_{q_i q'_i}^{M''}$ an $SU(3)_f$ flavor matrix corresponding to quark q_i changing to quark q'_i with emission of meson M'' (Fig.1). The second term in Eq.(10) is written in an unsymmetrized form since by commuting $\boldsymbol{\sigma} \cdot \mathbf{p}$ with $e^{-i\mathbf{k} \cdot \mathbf{r}}$ one obtains a term proportional to $\boldsymbol{\sigma} \cdot \mathbf{k} e^{-i\mathbf{k} \cdot \mathbf{r}}$ which can be absorbed in g . To eq.(10), one must also add the contribution from antiquarks

$$\mathcal{H}'' = X_{\bar{q}_j \bar{q}'_j}^{M''} \left[-g (\boldsymbol{\sigma}_j \cdot \mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{r}_j} + h (\boldsymbol{\sigma}_j \cdot \mathbf{p}_j) e^{-i\mathbf{k} \cdot \mathbf{r}_j} \right]. \quad (11)$$

All two-body decays are thus given in terms of two parameters h and g . The form (10) is common to both the elementary-emission model and the pair creation model of strong meson decays [9], the only difference being the values of the coefficients g and h . We assume that these decay constants are flavor independent. Although our results can be easily generalized to the case in which the decay constants of u ,

d and s are different, we prefer, for the purpose of the present article, to keep the transition operator in its simplest possible form. All flavor dependence is thus in the matrices X .

The decay widths are computed from

$$\Gamma(M \longrightarrow M' + M'') = \frac{k}{2\pi} \frac{E_{M'}}{E_M} |\langle M' | \mathcal{H} | M \rangle|^2, \quad (12)$$

where $\mathcal{H} = \mathcal{H}' + \mathcal{H}''$ and $E_{M'}$ and E_M are the total energy of the final meson and the mass of the initial meson. The ratio $\frac{E_{M'}}{E_M}$ can be rewritten in a frame independent way as a function of the masses of mesons appearing in eq.(1):

$$\chi = \frac{E_{M'}}{E_M} = \frac{1}{2} + \frac{m_{M'}^2 - m_{M''}^2}{2m_M^2}. \quad (13)$$

M'' is taken to be the lighter of the two mesons in the final state. If the two mesons M' and M'' are identical, an extra factor of $\frac{1}{2}$ is included. The operators (10) and (11) are expressed in the quark basis $|q_i \bar{q}_j\rangle$, but can be easily reexpressed in the $SU(3)_f$ basis by writing X in $SU(3)_f$ tensorial notation. In either case, while in the evaluation of the electromagnetic decays [8] the flavor part was trivial and all the complication was in the space-spin part, in the case of strong decays both parts require heavy algebra. Explicit expressions for the matrix X in the quark basis are given in Ref. [5], and they can be used directly for the evaluation of the matrix elements in (12). If instead $SU_f(3)$ wave functions are used, the evaluation of the matrix elements in (12) requires the knowledge of the $SU_f(3)$ and $SU_I(2)$ isoscalar factors

$$\left(\begin{array}{cc|c} M' & M'' & M \\ I_1 Y_1 & I_2 Y_2 & I Y \end{array} \right) \left(\begin{array}{cc|c} I_1 & I_2 & I \\ I_{z1} & I_{z2} & I_z \end{array} \right). \quad (14)$$

These are given in Ref. [10].

We have used both methods to evaluate the 25 strong decay widths shown in Table II. The use of both methods provides us with a check of the correctness of the results (the two methods differ only by an overall normalization factor). The computed widths are written in terms of some kinematic factors, the spin-flavor part and the form factors $F_i(\nu, k)$, $i = 1, \dots, 5$. The form factors F_i contain all the information of the hadronic structure: they depend on the momentum of the

emitted pseudoscalar meson k and on the ratio $\nu = m_{q(\bar{q})}/(m_q + m_{\bar{q}})$, where m_q and $m_{\bar{q}}$ are the quark and antiquark masses of the decaying meson. This dependence naturally arises in the calculation of the radial part of the matrix elements. The expressions in Table II include the sum and averaging over the components of the final and initial states, respectively. The calculations of the space part of the matrix elements, $e^{-i\mathbf{k}\cdot\mathbf{r}}$ for the first term and $\mathbf{p}e^{-i\mathbf{k}\cdot\mathbf{r}}$ for the second term, are done using the algebraic method discussed in Sect.II of Ref. [8], which is not repeated here. The relevant matrix elements are tabulated in Appendix A of Ref. [8].

C. Form factors

The widths in Table II are given in terms of form factors $F_i(k)$. By using algebraic methods, as discussed in Ref. [8], these form factors can be evaluated in closed form in three different situations:

- (I) $U(3)$: the $U(3)$ form factors are given in Table IIIa. They are all combinations of exponentials $\exp(-\alpha k^2)$, and polynomials in k^2 . These form factors are the form factors of the non-relativistic harmonic oscillator quark model or variations of it [4, 5].
- (II) $SO(4)$: the $SO(4)$ form factors are given in Table IIIb. They are combinations of spherical Bessel functions. These are the form factors of a rigid string with quarks sitting at its ends.
- (III) $SO(4)^*$: a more realistic case is that in which the meson is emitted from a string whose length is given by the probability distribution $\beta \exp(-2\beta/a)$. This produces form factors with a power law behaviour for large k . The corresponding expressions are obtained by replacing, in the $SO(4)$ form factors, the spherical Bessel functions $j_l(\beta k\nu)$ by

$$\tilde{j}_l(ak\nu) = \frac{\int_0^\infty d\beta (\beta e^{-2\beta/a}) j_l(\beta k\nu)}{\int_0^\infty d\beta (\beta e^{-2\beta/a})}. \quad (15)$$

The integrated functions \tilde{j}_l are shown in Table IIIc. This situation is denoted here by the $SO(4)^*$.

The three situations (I), (II), and (III) represent three extreme situations which encompass a large variety of hadronic structure models. The form factors of each situations contain one parameter (α , β or a) characterizing the average size of the mesons.

3 Analysis of Experimental Data

The results of Table II can be used to analyze the experimental data. A least square fit to the available data gives the values of the parameters shown in Table IV. With these values one can calculate the strong decay widths shown in Table V. When several final charge states are possible, the appropriate isospin factors have been included going from Table II to Table V. Also, in calculating the values of Table V the following mixing angles have been used

$$\theta_P = -23^\circ, \quad \theta_V = 38^\circ, \quad \theta_T = 26^\circ. \quad (16)$$

These values have been kept fixed. The value of the pseudoscalar mixing angle $\theta_P = -23^\circ$ is consistent with that determined in Ref. [7] from a fit to the meson masses. The value of θ_V , when converted to the notation of Ref. [8], is 2.7° , which is somewhat different from the value 4.3° of this reference. Finally, the value of θ_T for the tensor mesons is the same as reported in [1].

In order to display clearly the situation, we show in Figs.2–6 a comparison between the experimental data and the calculations. One can see that the agreement between calculated values and experiment is in most cases good, with few exceptions discussed in subsections A and B below. One can also see that, within the range of momenta tested by the decays of Table II, the three classes of form factors produce similar results. This is part of the reason why the harmonic oscillator quark model, although in clear contradiction with experiment for large k^2 , provides a reasonably good description of known decay widths.

In view of the fact that the calculated values agree with the experimental values on the average within a factor of two or better, one can address specific problems related to meson spectroscopy.

A. Decays of K_1^+ and the tensor force

Table V and Figs.2 and 6 show that two of the calculated decays which are in bad agreement with experiment are:

$$\begin{aligned} K_1^+(1270) &\longrightarrow \rho^0 K^+, \\ K_1^+(1400) &\longrightarrow \rho^0 K^+. \end{aligned} \quad (17)$$

The quantum numbers of $K_1^+(1270)$ and $K_1^+(1400)$ are 1P_1 and 3P_1 respectively. The calculated decay widths can be brought into better agreement with experiment by mixing of the two states,

$$\begin{aligned} |K_1^+(1270)\rangle &= \cos \varphi |^1P_1\rangle + \sin \varphi |^3P_1\rangle, \\ |K_1^+(1400)\rangle &= -\sin \varphi |^1P_1\rangle + \cos \varphi |^3P_1\rangle. \end{aligned} \quad (18)$$

The mixing between 1P_1 and 3P_1 can be produced only by a tensor force. The decays (17) provide therefore further evidence for the occurrence of a tensor force, similar to the evidence obtained from baryons (mixing of $N(1535)$ and $N(1650)$). Since one is looking for clues for the correctness of QCD in the non-perturbative regime, the decays (17) appear to confirm the occurrence of a tensor force

$$S_{12} = A [T_s^{(2)} \otimes T_{\mathcal{R}}^{(2)}], \quad (19)$$

where $T_s^{(2)}$ is an operator of rank 2 acting on the spin variables and $T_{\mathcal{R}}^{(2)}$ an operator of rank 2 acting on the space variables. The tensor force can be formally derived in QCD from one-gluon exchange.

The value $\varphi = 46.8^\circ$ gives

$$\begin{aligned} \Gamma(K_1(1270)) \rightarrow \rho K &= 15.3 \text{ MeV}, \\ \Gamma(K_1(1400)) \rightarrow \rho K &= 5.2 \text{ MeV}, \end{aligned} \quad (20)$$

still not in complete agreement with experiment (37.8 ± 13.8 and 5.2 ± 5.2 , respectively), but in much better agreement than the unmixed values. The particular value of φ has been computed using the results of the $U(3)$ fit, but similar results hold in the $SO(4)$ and $SO(4)^*$ symmetry.

B. Decays of $f_2(1270)$, $f'_2(1525)$ and glueballs

Table V shows that the only other calculated decay width which is in bad agreement with experiments is

$$f'_2(1525) \longrightarrow \pi\pi. \quad (21)$$

The decays of f_2 and f'_2 into $K\bar{K}$, $\eta\eta$, $\pi\pi$ can be used to study admixtures of glueballs components into $q\bar{q}$ states. We shall readdress this question in a later publication where the mixing of $f_2(1270)$, $f'_2(1525)$ with the glueball candidate $f_2(1720)$ will be analyzed in great detail. Here we only stress that the anomalously large deviation $\Gamma(f'_2 \rightarrow \pi\pi)_{calc}/\Gamma(f'_2 \rightarrow \pi\pi)_{exp} \simeq 10$ seems to indicate that other components ($q^2\bar{q}^2$ or gluonic) are present in the region of $f_2(1270)$ (the calculated width of the $f'_2 \rightarrow \pi\pi$ decay could be brought into better agreement with experiment by using a different value for the tensor mixing angle, $\theta_T \simeq 32.7^\circ$, but this mixing angle would produce a bad agreement of other f_2 and f'_2 decays, especially for $f_2 \rightarrow \eta\eta$).

C. Decays of $\phi(1020)$ and tests of kinematics

The two decays

$$\begin{aligned} \phi &\longrightarrow K^+ K^- \\ \phi &\longrightarrow K_L^0 K_S^0 \end{aligned} \quad (22)$$

provide a test of the kinematics used in the calculation. The matrix elements for the two processes are identical (and for this reason the decay $\phi \rightarrow K_L^0 K_S^0$ is not shown in Tables II and V), while the values of k are slightly different, $k = 127$ MeV/c and $k = 110$ MeV/c respectively. The experimental ratio

$$\frac{\Gamma(\phi \longrightarrow K^+ K^-)}{\Gamma(\phi \longrightarrow K_L^0 K_S^0)} = (1.43 \pm 0.06) \quad (23)$$

can be compared with that calculated in (I), (II) and (III):

$$1.534 \quad (I), \quad 1.535 \quad (II), \quad 1.531 \quad (III). \quad (24)$$

The agreement is good, and practically the same in all three situations. This is an important point, because for decays of relativistic particles it is not at all obvious

what kinematic factors should be used in connection with the transition operator \mathcal{H} of Eqs.(10)-(11).

We note in passing that, in the early days of hadronic spectroscopy [11], the widths were calculated by introducing a form factor $|k^2/(k^2 + R^{-2})|^l$ and a kinematic factor k/M , i.e.

$$\Gamma \propto \left| \frac{k^2}{k^2 + R^{-2}} \right|^l \frac{k}{M}, \quad (25)$$

where M is the mass of the decaying meson, R^{-1} a measure of the size of the system ($R^{-1} = 350 \text{ MeV/c}$) and l is the angular momentum of the decaying particle. The kinematic factor and form factor give, for the ratio (23), 1.52, also in agreement with experiment.

D. Decays of $\omega(783)$ and isospin mixing

Although not directly related to the results of Table V, we comment briefly on another problem of interest which can be studied with strong decays: isospin mixing. The decay $\omega \rightarrow \pi\pi$ (not shown in Table V) is forbidden by G-parity. However, a small mixing of ω^0 with ρ^0 will allow the decay to go. From the observed decay width

$$\Gamma(\omega^0 \rightarrow \pi^+\pi^-) = (186.3 \pm 27.5) \text{ keV}, \quad (26)$$

one finds

$$\begin{aligned} |\rho^0\rangle &= \cos 2.0^\circ |I=1\rangle + \sin 2.0^\circ |I=0\rangle, \\ |\omega^0\rangle &= -\sin 2.0^\circ |I=1\rangle + \cos 2.0^\circ |I=0\rangle. \end{aligned} \quad (27)$$

The value of the mixing angle 2.0° is somewhat smaller than the value 6.8° determined from the analysis of the radiative decays $\rho^0 \rightarrow \gamma\pi^0$, $\rho^\pm \rightarrow \gamma\pi^\pm$ and $\omega^0 \rightarrow \gamma\pi^0$ [8].

4 Predictions

The parameterization of the strong decay widths described here allows one to make predictions for unknown widths within a factor of 2 or better. These predictions

can in turn be used to extract “new” physics.

A. Decays of $a_0(^3P_0)$ and missing states

As an example we address here two problems: the so-called “problem of missing states” and the nature of $a_0(980)$. We use the method of Sect. 2 to calculate the decay matrix of $a_0(^3P_0)$, $a_1(^3P_1)$ and $a_2(^3P_2)$ into $\rho\pi$, $\eta\pi$, $K\bar{K}$. The allowed decays of a_2 and a_1 are already given in Table II. The decays of a_0 are shown in Table VI. The predicted decay matrix is shown in Table VII, where we report the results of the $U(3)$ symmetry (the $SO(4)$ and $SO(4)^*$ situations have similar qualitative behaviour). We note that the decays which are experimentally seen are in excellent agreement with calculations, and that the fact that the transition operator of Sect. 2 gives

$$\Gamma(a_1(1260) \longrightarrow \eta\pi) = 0 \quad \text{and} \quad \Gamma(a_1(1260) \longrightarrow K\bar{K}) = 0, \quad (28)$$

is in agreement with the non-observation of these decays. Most importantly, we find that the decays of the a_0 give a total width of about 420–940 MeV, for a_0 masses in the range 1200–1400 MeV with two branches $\eta\pi$ and $K\bar{K}$ (since $\Gamma(a_0(^3P_0) \rightarrow \rho\pi) = 0$ with the transition operator of Sect.2). This result suggests strongly that the $a_0(980)$, which has a total decay width of 57 ± 11 MeV, is *not* the $q\bar{q}$ state 3P_0 . Furthermore, the width of the $a_0(^3P_0)$ state is so large that it might escape detection. This indeed may be one of the reason why some states predicted by models of hadron structure are either not seen or marginally seen (missing states). The combination of the result of Ref. [7], which predicts the state $a_0(^3P_0)$ at 1273 MeV and the present paper, strongly suggest that the state $a_0(1320)$ reported at the Hadron 89 Conference and omitted from the Summary Tables of Ref. [1] is a good candidate for the missing $a_0(^3P_0)$ state.

5 Conclusions

We have presented here a reanalysis of the strong decays of $q\bar{q}$ mesons similar in spirit to the earlier analyses in terms of flavor $SU_f(3)$ symmetry [11], but where

symmetries are also used to deal with the space part of the hadron wavefunctions. In particular, for mesons, we have used the spectrum generating algebra $\mathcal{G} = U(4) \otimes SU_s(2) \otimes SU_f(3) \otimes SU_c(3)$, and considered two branchings of $U(4)$, into $U(3)$ and $SO(4)$. The use of $U(4)$ provides us with explicit expressions for the form factors, and thus allows us to compute analytically *all* decay widths.

We note that the results of Table V show that strong decay widths are not much dependent on models of hadronic structure ($U(3)$, $SO(4)$ and $SO(4)^*$) but they depend almost exclusively on the spin-flavor part of the meson wave functions. The parameterization of the decay operator (10) and (11) appears to give all decay widths within a factor of 2.

Having constructed the formalism for masses [7], electromagnetic [8], weak [12] and strong decays, we are now in a position to analyze any number of Ref. [1], related to these quantities. Deviations from symmetry parameterizations can be used to extract “new” physics. Particularly important in meson spectroscopy are the searches for gluonic components and multiquark configurations ($q^2\bar{q}^2, \dots$). We believe we have now a method in which this search can be done in a somewhat quantitative way. We also find it important to test the accuracy of the predictions of this article and of Ref. [8], by doing new experiments. New hadronic facilities, such as the ϕ factory, presently under construction at Frascati, Italy, may help in this respect, especially in the study of the decays of a_0 . Values of the calculated decay widths for any two-body strong decay can be obtained from us upon request.

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Figure and Table Captions

Fig.1 Elementary meson emission: part (a) refers to the operator \mathcal{H}'' , part (b) to \mathcal{H}' .

Figs.2 Comparison between calculated and experimental values of the form factor $|F_1(k)|$ as a function of k . The experimental points are the values of $\sqrt{\Gamma_{exp}}$ divided by the appropriate factors appearing in Table II. The theoretical curves are the form factors $|F_1(1/2, k)|$ with the values of g and h (or h') given in Table IV (full line, $U(3)$; dashed, $SO(4)$; dotted, $SO(4)^*$).

Fig.3 Same as Fig.2, but for $|F_2(k)|$. The experimental value at $k=0.36$ which is in bad agreement with calculations is that of $K_1(1270) \rightarrow \rho K$ discussed in Sect.3.A.

Fig.4 Same as Fig.2, but for $|F_3(k)|$. The experimental value at $k=1.52$ (in bad agreement with predictions) is that of $K_1(1400) \rightarrow \rho K$ discussed in Sect.3.A.

Fig.5 Same as Fig.2, but for $|F_4(k)|$. The experimental value at $k=3.15$ is that of $f'_2(1525) \rightarrow \pi\pi$ discussed in Sect.3.B.

Fig.6 Same as Fig.2, but for $|F_5(k)|$.

Table I Quantum numbers assignments of $q\bar{q}$ states in $U(3)$ and $SO(4)$.

Table II Analytic strong decay widths for selected light mesons. The pseudoscalar, vector and tensor mixing angles are denoted by θ_P , θ_V and θ_T . m indicates the $u-d$ mass (~ 250 MeV), m_s the strange quark mass (~ 400 MeV) and χ is defined in eq.(13).

Table III Form factors appearing in Table II. The parameter h' in (b) and (c) is equal to $h\zeta$, where ζ is the scale of the momenta [8]. The phase conventions for the $SO(4)$ wave functions is the same as for $U(3)$.

Table IV Values of the best fit parameters. g and h are in fm, h' is dimensionless.

Table V Comparison between the experimental [1] and calculated decay widths.
All widths are in MeV, k is in fm^{-1} .

Table VI Analytic strong decay widths for $a_0(^3P_0)$ decays.

Table VII Calculated strong decays of a_0 , a_1 and a_2 mesons. Since the mass of a_0 is not known, the values in the table are for $m_{a_0}=1200\text{--}1400$ MeV. All values are in MeV.

Table I

Meson	J^{PC}	$U(3)$ symmetry			$SO(4)$ symmetry		
		n	L	S	v	L	S

π family

π	0^{-+}	0	0	0	0	0	0
$\rho(770)$	1^{--}	0	0	1	0	0	1
$b_1(1235)$	1^{+-}	1	1	0	0	1	0
$a_1(1260)$	1^{++}	1	1	1	0	1	1
$a_2(1320)$	2^{++}	1	1	1	0	1	1
$\rho(1450)$	1^{--}	2	0	1	1	0	1
$\pi_2(1670)$	2^{-+}	2	2	0	0	2	0
$\rho_3(1690)$	3^{--}	2	2	1	0	2	1
$\rho(1700)$	1^{--}	2	2	1	0	2	1

K family

K	0^{-}	0	0	0	0	0	0
$K^*(892)$	1^{-}	0	0	1	0	0	1
$K_1(1270)$	1^{+}	1	1	0	0	1	0
$K^*(1370)$	1^{-}	2	0	1	1	0	1
$K_1(1400)$	1^{+}	1	1	1	0	1	1
$K_0^*(1430)$	0^{+}	1	1	1	0	1	1
$K_2^*(1430)$	2^{+}	1	1	1	0	1	1
$K^*(1680)$	1^{-}	2	2	1	0	2	1
$K_2(1770)$	2^{-}	2	2	1	0	2	1
$K_3^*(1780)$	3^{-}	2	2	1	0	2	1
$K_4^*(2045)$	4^{+}	3	3	1	0	3	1

Table I (continued)

		$U(3)$ symmetry			$SO(4)$ symmetry		
Meson	J^{PC}	n	L	S	v	L	S
η family							
η	0^{-+}	0	0	0	0	0	0
$\eta'(958)$	0^{-+}	0	0	0	0	0	0
$\omega(783)$	1^{--}	0	0	1	0	0	1
$\phi(1020)$	1^{--}	0	0	1	0	0	1
$h_1(1170)$	1^{+-}	1	1	0	0	1	0
$f_2(1270)$	2^{++}	1	1	1	0	1	1
$f_1(1285)$	1^{++}	1	1	1	0	1	1
$\eta(1295)$	0^{-+}	2	0	0	1	0	0
$\omega(1390)$	1^{--}	2	0	1	1	0	1
$f_1(1510)$	1^{++}	1	1	1	0	1	1
$f_2'(1525)$	2^{++}	1	1	1	0	1	1
$\omega(1600)$	1^{--}	2	2	1	0	2	1
$\omega_3(1600)$	3^{--}	2	2	1	0	2	1
$\phi(1680)$	1^{--}	2	0	1	1	0	1
$\phi_3(1850)$	3^{--}	2	2	1	0	2	1
$f_4(2050)$	4^{++}	3	3	1	0	3	1

Table II

	${}^3S_1 \longrightarrow {}^1S_0$
$\Gamma_{\rho^0(770) \rightarrow \pi^- \pi^+}$	$= \frac{k}{16\pi} F_1(1/2, k) ^2$
$\Gamma_{\phi(1020) \rightarrow K^- K^+}$	$= \frac{3k}{64\pi} \cos^2 \theta_V F_1(1/2, k) ^2$
$\Gamma_{K^{*0}(892) \rightarrow \pi^- K^+}$	$= \frac{k}{16\pi} \chi F_1(\frac{m}{m+m_s}, k) ^2$
	${}^3S_1 \longrightarrow {}^3S_1$
$\Gamma_{\phi(1020) \rightarrow \rho^- \pi^+}$	$= \frac{k}{6\pi} \chi (\sin \theta_V - \frac{\cos \theta_V}{\sqrt{2}})^2 F_1(1/2, k) ^2$
	${}^1P_1 \longrightarrow {}^3S_1$
$\Gamma_{b_1^+(1235) \rightarrow \omega \pi^+}$	$= \frac{k}{12\pi} \chi (\frac{\sin \theta_V}{\sqrt{2}} + \cos \theta_V)^2 F_2(1/2, k) ^2$
$\Gamma_{K_1^+(1270) \rightarrow \rho^0 K^+}$	$= \frac{k}{32\pi} \chi F_2(\frac{m}{m+m_s}, k) ^2$
$\Gamma_{K_1^+(1270) \rightarrow K^{0*} \pi^+}$	$= \frac{k}{16\pi} \chi F_2(\frac{m_s}{m+m_s}, k) ^2$
	${}^3P_1 \longrightarrow {}^3S_1$
$\Gamma_{a_1^0(1260) \rightarrow \rho^- \pi^+}$	$= \frac{k}{8\pi} \chi F_3(1/2, k) ^2$
$\Gamma_{K_1^+(1400) \rightarrow K^{0*} \pi^+}$	$= \frac{k}{16\pi} \chi F_3(\frac{m_s}{m+m_s}, k) ^2$
$\Gamma_{K_1^+(1400) \rightarrow \rho^0 K^+}$	$= \frac{k}{32\pi} \chi F_3(\frac{m}{m+m_s}, k) ^2$

Table II (continued)

${}^3P_2 \longrightarrow {}^1S_0$	
$\Gamma_{a_2^+(1320) \rightarrow \eta \pi^+}$	$= \frac{k}{20\pi} \chi \left(\sin \theta_P - \frac{\cos \theta_P}{\sqrt{2}} \right)^2 F_4(1/2, k) ^2$
$\Gamma_{a_2^0(1320) \rightarrow K^- K^+}$	$= \frac{3k}{320\pi} F_4(1/2, k) ^2$
$\Gamma_{K_2^{*0}(1430) \rightarrow \pi^- K^+}$	$= \frac{3k}{80\pi} \chi \left F_4\left(\frac{m}{m+m_s}, k\right) \right ^2$
$\Gamma_{K_2^{*0}(1430) \rightarrow \eta K^0}$	$= \frac{k}{160\pi} \chi \left \left(\sin \theta_P + \sqrt{2} \cos \theta_P \right) F_4\left(\frac{m_s}{m+m_s}, k\right) + \left(\sin \theta_P - \frac{\cos \theta_P}{\sqrt{2}} \right) F_4\left(\frac{m}{m+m_s}, k\right) \right ^2$
$\Gamma_{f_2'(1525) \rightarrow K^- K^+}$	$= \frac{k}{160\pi} \left(\frac{\cos \theta_T}{\sqrt{2}} + 2 \sin \theta_T \right)^2 F_4(1/2, k) ^2$
$\Gamma_{f_2'(1525) \rightarrow \eta \eta}$	$= \frac{1}{2} \frac{k}{180\pi} \left \left\{ 4 \left(\sin \theta_P - \frac{\cos \theta_P}{\sqrt{2}} \right) \left(\sin \theta_T - \frac{\cos \theta_T}{\sqrt{2}} \right) \left(-\sin \theta_P - \frac{\cos \theta_P}{4} \right) + \right. \right.$ $\left. 2 \left(\sin \theta_P + \sqrt{2} \cos \theta_P \right) \left(\sin \theta_T + \sqrt{2} \cos \theta_T \right) \left(\frac{\cos \theta_P}{2} - \sin \theta_P \right) \right\} F_4(1/2, k) \right ^2$
$\Gamma_{f_2'(1525) \rightarrow \pi^- \pi^+}$	$= \frac{k}{40\pi} \left(\sin \theta_T - \frac{\cos \theta_T}{\sqrt{2}} \right)^2 F_4(1/2, k) ^2$
$\Gamma_{f_2(1270) \rightarrow \pi^- \pi^+}$	$= \frac{k}{40\pi} \left(\frac{\sin \theta_T}{\sqrt{2}} + \cos \theta_T \right)^2 F_4(1/2, k) ^2$
$\Gamma_{f_2(1270) \rightarrow K^- K^+}$	$= \frac{k}{160\pi} \left(\frac{\sin \theta_T}{\sqrt{2}} - 2 \cos \theta_T \right)^2 F_4(1/2, k) ^2$
$\Gamma_{f_2(1270) \rightarrow \eta \eta}$	$= \frac{1}{2} \frac{k}{180\pi} \left \left\{ 4 \left(\frac{\sin \theta_P}{\sqrt{2}} + \cos \theta_P \right) \left(\frac{\sin \theta_T}{\sqrt{2}} + \cos \theta_T \right) \left(-\sin \theta_P - \frac{\cos \theta_P}{4} \right) + \right. \right.$ $\left. 2 \left(\sqrt{2} \sin \theta_P - \cos \theta_P \right) \left(\sqrt{2} \sin \theta_T - \cos \theta_T \right) \left(\frac{\cos \theta_P}{2} - \sin \theta_P \right) \right\} F_4(1/2, k) \right ^2$
${}^3P_0 \longrightarrow {}^1S_0$	
$\Gamma_{K_0^{*0}(1430) \rightarrow \pi^- K^+}$	$= \frac{3k}{16\pi} \chi \left F_5\left(\frac{m}{m+m_s}, k\right) \right ^2$
${}^3P_2 \longrightarrow {}^3S_1$	
$\Gamma_{a_2^+(1320) \rightarrow \rho^0 \pi^+}$	$= \frac{3k}{40\pi} \chi \frac{3}{2} F_4(1/2, k) ^2$
$\Gamma_{K_2^{*0}(1430) \rightarrow K^{*+} \pi^-}$	$= \frac{3k}{80\pi} \chi \frac{3}{2} F_4\left(\frac{m_s}{m+m_s}, k\right) ^2$
$\Gamma_{K_2^{*0}(1430) \rightarrow \rho^- K^+}$	$= \frac{3k}{80\pi} \chi \frac{3}{2} F_4\left(\frac{m}{m+m_s}, k\right) ^2$
$\Gamma_{K_2^{*0}(1430) \rightarrow \omega K^0}$	$= \frac{k}{80\pi} \chi \frac{3}{2} \left \left(\sqrt{2} \sin \theta_V - \cos \theta_V \right) F_4\left(\frac{m_s}{m+m_s}, k\right) + \left(\frac{\sin \theta_V}{\sqrt{2}} + \cos \theta_V \right) F_4\left(\frac{m}{m+m_s}, k\right) \right ^2$

Table III

(a) $U(3)$ symmetry

$$F_1(\nu, k) = \left[g + \frac{\nu}{2} h \right] k \exp\left(-\frac{\alpha^2 \nu^2 k^2}{4}\right)$$

$$F_2(\nu, k) = \frac{i}{\sqrt{2}} \left\{ \left| \frac{h}{\alpha} - k^2 \nu \alpha \left[g + \frac{\nu}{2} h \right] \right|^2 + \frac{2h^2}{\alpha^2} \right\}^{1/2} \exp\left(-\frac{\alpha^2 \nu^2 k^2}{4}\right)$$

$$F_3(\nu, k) = i \left\{ \frac{h^2}{\alpha^2} + 2 \left| -\frac{h}{\alpha} + \frac{1}{2} k^2 \nu \alpha \left[g + \frac{\nu}{2} h \right] \right|^2 \right\}^{1/2} \exp\left(-\frac{\alpha^2 \nu^2 k^2}{4}\right)$$

$$F_4(\nu, k) = \frac{i}{\sqrt{3}} k^2 \nu \alpha \left[g + \frac{\nu}{2} h \right] \exp\left(-\frac{\alpha^2 \nu^2 k^2}{4}\right)$$

$$F_5(\nu, k) = \frac{i}{\sqrt{6}} \left\{ k^2 \nu \alpha \left[g + \frac{\nu}{2} h \right] - \frac{3h}{\alpha} \right\} \exp\left(-\frac{\alpha^2 \nu^2 k^2}{4}\right)$$

(b) $SO(4)$ symmetry

$$F_1(\nu, k) = g k j_0(k\beta\nu) + h' j_1(k\beta\nu)$$

$$F_2(\nu, k) = i \left\{ \left| g k \sqrt{3} j_1(k\beta\nu) - \frac{h'}{\sqrt{3}} [j_0(k\beta\nu) - 2 j_2(k\beta\nu)] \right|^2 + \frac{2}{3} h'^2 |j_0(k\beta\nu) + j_2(k\beta\nu)|^2 \right\}^{1/2}$$

$$F_3(\nu, k) = i \left\{ \frac{2}{3} h'^2 |j_0(k\beta\nu) + j_2(k\beta\nu)|^2 + \left| g k \sqrt{3} j_1(k\beta\nu) - \frac{h'}{\sqrt{3}} [2 j_0(k\beta\nu) - j_2(k\beta\nu)] \right|^2 \right\}^{1/2}$$

$$F_4(\nu, k) = i \sqrt{2} [-g k j_1(k\beta\nu) - h' j_2(k\beta\nu)]$$

$$F_5(\nu, k) = i [g k j_1(k\beta\nu) - h' j_0(k\beta\nu)]$$

(c) $SO(4)^*$ symmetry: replace j_l in part (b) by \tilde{j}_l .

$$\tilde{j}_0(kav) = \frac{1}{1+Q^2}$$

$$\tilde{j}_1(kav) = \frac{1}{Q(1+Q^2)} \{[Q + \frac{1}{Q}] \text{atan} Q - 1\}$$

$$\tilde{j}_2(kav) = \frac{2}{1+Q^2} \{1 - \frac{3}{2Q^2} ([Q + \frac{1}{Q}] \text{atan} Q - 1)\}$$

where $Q = \frac{a}{2}k\nu$

Table IV

	g	h or h'	size parameter (fm)
$U(3)$	2.87	-0.79	$\alpha = 0.49$
$SO(4)$	2.83	-1.99	$\beta = 0.54$
$SO(4)^*$	2.96	-1.91	$a = 0.65$

Table V

Decay	Exp. value	$U(3)$	$SO(4)$	$SO(4)^*$	k
$\rho(770) \rightarrow \pi\pi$	151.5 ± 1.2	152.0	151.8	150.1	1.81
$\phi(1020) \rightarrow K^+ K^-$	2.18 ± 0.06	3.44	3.39	3.60	0.64
$K^*(892) \rightarrow \pi K$	50.5 ± 0.6	48.5	47.8	49.8	1.47
$\phi(1020) \rightarrow \rho\pi$	0.57 ± 0.04	0.46	0.46	0.48	0.93
$b_1(1235) \rightarrow \omega\pi$	155 ± 8	82	80	83	1.77
$K_1(1270) \rightarrow \rho K$	37.8 ± 13.8	5.2	5.2	4.8	0.36
$K_1(1270) \rightarrow K^*\pi$	14 ± 8	51	51	51	1.52
$a_1(1260) \rightarrow \rho\pi$	~ 400	341	340	341	1.92
$K_1(1400) \rightarrow K^*\pi$	164 ± 23	165	170	163	2.03
$K_1(1400) \rightarrow \rho K$	5 ± 5	63	62	62	1.52
$a_2(1320) \rightarrow \eta\pi$	16 ± 2	33	30	34	2.71
$a_2(1320) \rightarrow K\bar{K}$	5.4 ± 1.1	7.7	6.9	8.5	2.21
$K_2^*(1430) \rightarrow \pi K$	54 ± 4	36	32	38	3.15
$K_2^*(1430) \rightarrow \eta K$	$0.15 \begin{pmatrix} +0.31 \\ -0.11 \end{pmatrix}$	0.09	0.14	0.05	2.48
$f_2'(1525) \rightarrow K\bar{K}$	54 ± 9	43	41	44	2.94
$f_2'(1525) \rightarrow \eta\eta$	21 ± 5	21	20	22	2.68
$f_2'(1525) \rightarrow \pi\pi$	0.62 ± 0.20	8.94	9.25	8.06	3.80
$f_2(1270) \rightarrow \pi\pi$	$157 \begin{pmatrix} +22 \\ -19 \end{pmatrix}$	151	146	148	3.15
$f_2(1270) \rightarrow K\bar{K}$	8.5 ± 1.9	7.7	6.9	8.7	2.04
$f_2(1270) \rightarrow \eta\eta$	0.8 ± 0.3	0.4	0.3	0.5	1.64
$K_0^*(1430) \rightarrow \pi K$	267 ± 50	341	339	340	3.15
$a_2(1320) \rightarrow \rho\pi$	77 ± 6	51	45	57	2.13
$K_2^*(1430) \rightarrow K^*\pi$	27 ± 3	28	26	29	2.14
$K_2^*(1430) \rightarrow \rho K$	9.6 ± 1.3	3.5	2.9	4.2	1.69
$K_2^*(1430) \rightarrow \omega K$	3.16 ± 1.02	1.15	0.97	1.39	1.62

Table VI

$$\Gamma_{a_0^+ \rightarrow \eta \pi^+} = \frac{k}{4\pi} \chi \left[\sin \theta_P - \frac{\cos \theta_P}{\sqrt{2}} \right]^2 |F_5(1/2, k)|^2$$

$$\Gamma_{a_0^0 \rightarrow K^- K^+} = \frac{3k}{64\pi} |F_5(1/2, k)|^2$$

Table VII

	decay modes		
	$\rho \pi$	$\eta \pi$	$K \bar{K}$
a_2	51	33	8
a_1	341	0	0
a_0	0	335–700	86–240

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